

# The Cosmological Constant Problems\*

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## Abstract

The old cosmological constant problem is to understand why the vacuum energy is so small; the new problem is to understand why it is comparable to the present mass density. Several approaches to these problems are reviewed. Quintessence does not help with either; anthropic considerations offer a possibility of solving both. In theories with a scalar field that takes random initial values, the anthropic principle may apply to the cosmological constant, but probably to nothing else.

## 1. Introduction

There are now two cosmological constant problems. The old cosmological constant problem is to understand in a natural way why the vacuum energy density  $\rho_V$  is not very much larger. We can reliably calculate some contributions to  $\rho_V$ , like the energy density in fluctuations in the gravitational field at graviton energies nearly up to the Planck scale, which is larger than is observationally allowed by some 120 orders of magnitude. Such terms in  $\rho_V$  can be cancelled by other contributions that we can't calculate, but the cancellation then has to be accurate to 120 decimal places. The new cosmological constant problem is to understand why  $\rho_V$  is not only small, but also, as current Type Ia supernova observations seem to indicate,<sup>2</sup> of the same order of magnitude as the present mass density of the universe.

The efforts to understand these problems can be grouped into four general classes. The first approach is to imagine some scalar field coupled to gravity in such a way that  $\rho_V$  is automatically cancelled or nearly cancelled when the scalar field reaches its equilibrium value. In a review article over a decade ago<sup>3</sup> I gave a sort of 'no go' theorem, showing why such attempts

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<sup>2</sup> A. G. Riess *et al.*: *Astron. J.* **116**, 1009 (1998); P. M. Garnavich *et al.*: *Astrophys. J.* **509**, 74 (1998); S. Perlmutter *et al.*: *Astrophys. J.* **517**, 565 (1999).

<sup>3</sup>S. Weinberg: *Rev. Mod. Phys.* **61**, 1 (1989).

would not work without the need for a fine tuning of parameters that is just as mysterious as the problem we started with. I wouldn't claim that this is conclusive — other no-go theorems have been evaded in the past — but so far no one has found a way out of this one. The second approach is to imagine some sort of deep symmetry, one that is not apparent in the effective field theory that governs phenomena at accessible energies, but that nevertheless constrains the parameters of this effective theory so that  $\rho_V$  is zero or very small. I leave this to be covered in the talk by Edward Witten. In this talk I will concentrate on the third and fourth of these approaches, based respectively on the idea of quintessence and on versions of the anthropic principle.

## 2. Quintessence

The idea of quintessence<sup>4</sup> is that the cosmological constant is small because the universe is old. One imagines a uniform scalar field  $\phi(t)$  that rolls down a potential  $V(\phi)$ , at a rate governed by the field equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 , \quad (1)$$

where  $H$  is the expansion rate

$$H = \sqrt{\left(\frac{3}{8\pi G}\right) (\rho_\phi + \rho_M)} . \quad (2)$$

Here  $\rho_\phi$  is the energy density of the scalar field

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) , \quad (3)$$

while  $\rho_M$  is the energy density of matter and radiation, which decreases as

$$\dot{\rho}_M = -3H (\rho_M + p_M) , \quad (4)$$

with  $p_M$  the pressure of matter and radiation.

If there is some value of  $\phi$  (typically,  $\phi$  infinite) where  $V'(\phi) = 0$ , then it is natural that  $\phi$  should approach this value, so that it eventually changes only slowly with time. Meanwhile  $\rho_M$  is steadily decreasing, so that eventually the universe starts an exponential expansion with a slowly varying expansion

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<sup>4</sup>P. J. E. Peebles and B. Ratra: *Astrophys. J.* **325**, L17 (1988); B. Ratra and P. J. E. Peebles: *Phys. Rev.* **D 37**, 3406 (1988); C. Wetterich: *Nucl. Phys.* **B302**, 668 (1988).

rate  $H \simeq \sqrt{8\pi G V(\phi)}/3$ . The problem, of course, is to explain why  $V(\phi)$  is small or zero at the value of  $\phi$  where  $V'(\phi) = 0$ .

Recently this approach has been studied in the context of so-called ‘tracker’ solutions.<sup>5</sup> The simplest case arises for a potential of the form

$$V(\phi) = M^{4+\alpha} \phi^{-\alpha}, \quad (5)$$

where  $\alpha > 0$ , and  $M$  is an adjustable constant. If the scalar field begins at a value much less than the Planck mass and with  $V(\phi)$  and  $\dot{\phi}^2$  much less than  $\rho_M$ , then the field  $\phi(t)$  initially increases as  $t^{2/(2+\alpha)}$ , so that  $\rho_\phi$  decreases as  $t^{-2\alpha/(2+\alpha)}$ , while  $\rho_M$  is decreasing faster, as  $t^{-2}$ . (The existence of this phase is important, because the success of cosmic nucleosynthesis calculations would be lost if the cosmic energy density were not dominated by  $\rho_M$  at temperatures of order  $10^9$  °K to  $10^{10}$  °K.) Eventually a time is reached when  $\rho_M$  becomes as small as  $\rho_\phi$ , after which the character of the solution changes. Now  $\rho_\phi$  becomes larger than  $\rho_M$ , and  $\rho_\phi$  decreases more slowly, as  $t^{-2/(4+\alpha)}$ . The expansion rate  $H$  now goes as  $H \propto \sqrt{V(\phi)} \propto t^{-\alpha/(4+\alpha)}$ , so the Robertson–Walker scale factor  $R(t)$  grows almost exponentially, with  $\log R(t) \propto t^{4/(4+\alpha)}$ . In this approach, the transition from  $\rho_M$ -dominance to  $\rho_\phi$ -dominance is supposed to take place near the present time, so that both  $\rho_M$  and  $\rho_\phi$  are now both contributing appreciably to the cosmic expansion rate.

The nice thing about these tracker solutions is that the existence of a cross-over from an early  $\rho_M$ -dominated expansion to a later  $\rho_\phi$ -dominated expansion does not depend on any fine-tuning of the initial conditions. But it should not be thought that *either* of the two cosmological constant problems are solved in this way. Obviously, the decrease of  $\rho_\phi$  at late times would be spoiled if we added a constant of order  $m_{\text{Planck}}^4$  (or  $m_W^4$ , or  $m_e^4$ ) to the potential (5). What is perhaps less clear is that, even if we take the potential in the form (5) without any such added constant, we still need a fine-tuning to make the value of  $\rho_\phi$  at which  $\rho_\phi \approx \rho_M$  close to the *present* critical density  $\rho_{c0}$ . The value of the field  $\phi(t)$  at this crossover can easily be seen to be of the order of the Planck mass, so in order for  $\rho_\phi$  to be comparable to  $\rho_M$  at the present time we need

$$M^{4+\alpha} \approx (8\pi G)^{-\alpha/2} \rho_{c0} \approx (8\pi G)^{-1-\alpha/2} H_0^2. \quad (6)$$

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<sup>5</sup>I. Zlatev, L. Wang, and P. J. Steinhardt: Phys. Rev. Lett. **82**, 896 (1999); Phys. Rev. **D 59**, 123504 (1999).

Theories of quintessence offer no explanation why this should be the case. (An interesting suggestion has been made after Dark Matter 2000.<sup>6</sup>)

### 3. Anthropic Considerations

In several cosmological theories the observed big bang is just one member of an ensemble. The ensemble may consist of different expanding regions at different times and locations in the same spacetime,<sup>7</sup> or of different terms in the wave function of the universe.<sup>8</sup> If the vacuum energy density  $\rho_V$  varies among the different members of this ensemble, then the value observed by any species of astronomers will be conditioned by the necessity that this value of  $\rho_V$  should be suitable for the evolution of intelligent life.

It would be a disappointment if this were the solution of the cosmological constant problems, because we would like to be able to calculate all the constants of nature from first principles, but it may be a disappointment that we will have to live with. We have learned to live with similar disappointments in the past. For instance, Kepler tried to derive the relative distances of the planets from the sun by a geometrical construction involving Platonic solids nested within each other, and it was somewhat disappointing when Newton's theory of the solar system failed to constrain the radii of planetary orbits, but by now we have gotten used to the fact that these radii are what they are because of historical accidents. This is a pretty good analogy, because we do have an anthropic explanation why the planet on which we live is in the narrow range of distances from the sun at which the surface temperature allows the existence of liquid water: if the radius of our planet's orbit was not in this range, then we would not be here. This would not be a satisfying explanation if the earth were the only planet in the universe, for then the fact that it is just the right distance from the sun to allow water to be liquid on its surface would be quite amazing. But with nine planets in our solar system and vast numbers of planets in the rest of the universe, at different distances from their respective stars, this sort of anthropic explanation is just common sense. In the same way, an anthropic explanation of the value of  $\rho_V$  makes sense if and only if there is a very large

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<sup>6</sup> C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt: astro-ph/0004134.

<sup>7</sup> A. Vilenkin: Phys. Rev. **D 27**, 2848 (1983); A. D. Linde: Phys. Lett. **B175**, 395 (1986).

<sup>8</sup> E. Baum: Phys. Lett. **B133**, 185 (1984); S. W. Hawking: in *Shelter Island II – Proceedings of the 1983 Shelter Island Conference on Quantum Field Theory and the Fundamental Problems of Physics*, ed. by R. Jackiw *et al.* (MIT Press, Cambridge, 1985); Phys. Lett. **B134**, 403 (1984); S. Coleman: Nucl. Phys. **B 307**, 867 (1988).

number of big bangs, with different values for  $\rho_V$ .

The anthropic bound on a positive vacuum energy density is set by the requirement that  $\rho_V$  should not be so large as to prevent the formation of galaxies.<sup>9</sup> Using the simple spherical infall model of Peebles<sup>10</sup> to follow the nonlinear growth of inhomogeneities in the matter density, one finds an upper bound

$$\rho_V < \frac{500 \rho_R \delta_R^3}{729} \quad (7)$$

where  $\rho_R$  is the mass density and  $\delta_R$  is a typical fractional density perturbation, both taken at the time of recombination. This is roughly the same as requiring that  $\rho_V$  should be no larger than the cosmic mass density at the earliest time of galaxy formation, which for a maximum galactic redshift of 5 would be about 200 times the present mass density. This is a big improvement over missing by 120 orders of magnitude, but not good enough.

However, we would not expect to live in a big bang in which galaxy formation is just barely possible. Much more reasonable is what Vilenkin calls a *principle of mediocrity*,<sup>11</sup> which suggests that we should expect to find ourselves in a big bang that is typical of those in which intelligent life is possible. To be specific, if  $\mathcal{P}_{\text{a priori}}(\rho_V) d\rho_V$  is the *a priori* probability of a particular big bang having vacuum energy density between  $\rho_V$  and  $\rho_V + d\rho_V$ , and  $\mathcal{N}(\rho_V)$  is the average number of scientific civilizations in big bangs with energy density  $\rho_V$ , then the actual (unnormalized) probability of a scientific civilization observing an energy density between  $\rho_V$  and  $\rho_V + d\rho_V$  is

$$d\mathcal{P}(\rho_V) = \mathcal{N}(\rho_V) \mathcal{P}_{\text{a priori}}(\rho_V) d\rho_V . \quad (8)$$

We don't know how to calculate  $\mathcal{N}(\rho_V)$ , but it seems reasonable to take it as proportional to the number of baryons that wind up in galaxies, with an unknown proportionality factor that is independent of  $\rho_V$ . There is a complication, that the total number of baryons in a big bang may be infinite, and may also depend on  $\rho_V$ . In practice, we take  $\mathcal{N}(\rho_V)$  as the *fraction* of baryons that wind up in galaxies, which we can hope to calculate, and include the total baryon number as a factor in  $\mathcal{P}_{\text{a priori}}(\rho_V)$ .

The one thing that offers some hope of actually calculating  $d\mathcal{P}(\rho_V)$  is that  $\mathcal{N}(\rho_V)$  is non-zero in only a narrow range of values of  $\rho_V$ , values that are

<sup>9</sup>S. Weinberg: Phys. Rev. Lett. **59**, 2607 (1987).

<sup>10</sup>P. J. E. Peebles: Astrophys. J. **147**, 859 (1967).

<sup>11</sup>A. Vilenkin: Phys. Rev. Lett. **74**, 846 (1995); in *Cosmological Constant and the Evolution of the Universe*, ed. by K. Sato *et al.* (Universal Academy Press, Tokyo, 1996).

much smaller than the energy densities typical of elementary particle physics, so that  $\mathcal{P}_{\text{a priori}}(\rho_V)$  is likely to be constant within this range.<sup>12</sup> The value of this constant is fixed by the requirement that the total probability should be one, so

$$d\mathcal{P}(\rho_V) = \frac{\mathcal{N}(\rho_V) d\rho_V}{\int \mathcal{N}(\rho'_V) d\rho'_V}. \quad (9)$$

The fraction  $\mathcal{N}(\rho_V)$  of baryons in galaxies has been calculated by Martel, Shapiro and myself,<sup>13</sup> using the well-known spherical infall model of Gunn and Gott,<sup>14</sup> in which one starts with a fractional density perturbation that is positive within a sphere, and compensated by a negative fractional density perturbation in a surrounding spherical shell. The results are quite insensitive to the relative radii of the sphere and shell. Taking the shell thickness to equal the sphere's radius, the integrated probability distribution function for finding a vacuum energy less than or equal to  $\rho_V$  is

$$\begin{aligned} \mathcal{P}(\leq \rho_V) &\equiv \int_0^{\rho_V} d\mathcal{P} \\ &= 1 + (1 + \beta)e^{-\beta} \\ &+ \frac{1}{2 \ln 2 - 1} \int_{\beta}^{\infty} e^{-x} dx \left\{ -2\sqrt{\beta x} + \beta + 2x \ln \left[ \sqrt{\beta/x} + 1 \right] \right\} \end{aligned} \quad (10)$$

where

$$\beta \equiv \frac{1}{2\sigma^2} \left( \frac{729 \rho_V}{500 \rho_R} \right)^{2/3} \quad (11)$$

with  $\sigma$  the rms fractional density perturbation at recombination, and  $\rho_R$  the average mass density at recombination. The probability of finding ourselves in a big bang with a vacuum energy density large enough to give a present value of  $\Omega_V$  of 0.7 or less turns out to be 5% to 12%, depending on the assumptions used to estimate  $\sigma$ . In other words, the vacuum energy in our big bang still seems a little low, but not implausibly so. *These anthropic considerations can therefore provide a solution to both the old and the new cosmological constant problems*, provided of course that the underlying assumptions are valid. Related anthropic calculations have been carried out by several other authors.<sup>15</sup>

<sup>12</sup>S. Weinberg: in *Critical Dialogs in Cosmology*, ed. by N. Turok (World Scientific, Singapore, 1997).

<sup>13</sup>H. Martel, P. Shapiro, and S. Weinberg: *Astrophys. J.* **492**, 29 (1998).

<sup>14</sup>J. Gunn and J. Gott: *Astrophys. J.* **176**, 1 (1972).

<sup>15</sup>G. Efstathiou: *Mon. Not. Roy. Astron. Soc.* **274**, L73 (1995); M. Tegmark and M.

I should add that when anthropic considerations were first applied to the cosmological constant, counts of galaxies as a function of redshift<sup>16</sup> indicated that  $\Omega_\Lambda$  is  $0.1_{-0.4}^{+0.2}$ , and this was recognized to be too small to be explained anthropically. The subsequent discovery in studies of type Ia supernova distances and redshifts that  $\Omega_\Lambda$  is quite large does not of course prove that anthropic considerations are relevant, but it is encouraging.

Recently the assumptions underlying these calculations have been challenged by Garriga and Vilenkin.<sup>17</sup> They adopt a plausible model for generating an ensemble of big bangs with different values of  $\rho_V$ , by supposing that there is a scalar field  $\phi$  that initially can take values anywhere in a broad range in which the potential  $V(\phi)$  is very flat. Specifically, in this range

$$\left| \frac{V'(\phi)}{V(\phi)} \right| \ll \sqrt{8\pi G} \quad \text{and} \quad \left| \frac{V''(\phi)}{V(\phi)} \right| \ll 8\pi G. \quad (12)$$

It is also assumed that in this range  $V(\phi)$  is much less than the initial value of the energy density  $\rho_M$  of matter and radiation. For initial values of  $\phi$  in this range, the vacuum energy density  $\rho_\phi$  stays roughly constant while  $\rho_M$  drops to a value of order  $\rho_\phi$ . To see this, note that during this period the expansion rate behaved as  $H = \eta/t$ , with  $\eta = 2/3$  or  $\eta = 1/2$  during times of matter or radiation dominance, respectively. If we tentatively assume that  $\phi$  is roughly constant, then the field equation (1) gives

$$\dot{\phi} \simeq -\frac{t V'(\phi)}{1 + 3\eta}. \quad (13)$$

During the time that  $\rho_M \gg \rho_\phi$ , the ratio of the kinetic to the potential terms in Eq. (3) for  $\rho_\phi$  is

$$\frac{\dot{\phi}^2}{2V(\phi)} \simeq \frac{t^2 V'^2(\phi)}{2(1 + 3\eta)^2 V(\phi)} \ll \frac{8\pi G t^2 V(\phi)}{2(1 + 3\eta)^2} \simeq \frac{3\eta^2 V(\phi)}{2(1 + 3\eta)^2 \rho_M} \ll 1, \quad (14)$$

so  $\rho_\phi$  is dominated by the potential term. The fractional change in  $\rho_\phi$  until the time  $t_c$  when  $\rho_M$  becomes equal to  $\rho_\phi$  is then

$$\frac{|\Delta\rho_\phi|}{\rho_\phi} = \frac{1}{\rho_\phi} \left| \int_0^{t_c} V'(\phi) \dot{\phi} dt \right| \simeq \frac{V'^2(\phi) t_c^2}{2(1 + 3\eta)\rho_\phi} \approx \frac{V'^2(\phi)}{8\pi G \rho_\phi^2} \ll 1. \quad (15)$$

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J. Rees: *Astrophys. J.* **499**, 526 (1998); J. Garriga, M. Livio, and A. Vilenkin: *astro-ph/9906210*; S. Bludman: *astro-ph/0002204*.

<sup>16</sup>E. D. Loh: *Phys. Rev. Lett.* **57**, 2865 (1986).

<sup>17</sup>J. Garriga and A. Vilenkin: *astro-ph/9908115*.

Following this period,  $\rho_\phi$  becomes dominant, and the inequalities (12) ensure that the expansion becomes essentially exponential, just as in theories with the ‘tracker’ solutions discussed in the previous section. Hence in this class of models,  $V(\phi)$  plays the role of a constant vacuum energy, whose values are governed by the *a priori* probability distribution for the initial values of  $\phi$ . In particular, if one assumes that all initial values of  $\phi$  are equally probable, then the *a priori* distribution of the vacuum energy is

$$\mathcal{P}_{\text{a priori}}(V(\phi)) \propto \frac{1}{|V'(\phi)|} . \quad (16)$$

The point made by Garriga and Vilenkin was that, because  $V(\phi)$  is so flat, the field  $\phi$  can vary appreciably even when  $\rho_V \simeq V(\phi)$  is restricted to the very narrow anthropically allowed range of values in which galaxy formation is possible. They concluded that it would also be possible for the *a priori* probability (16) to vary appreciably in this range, which if true would require modifications in the calculation of  $\mathcal{P}(\leq \rho_V)$  described above. The potential they used as an example was

$$V(\phi) = V_1 + A(\phi/M) + B \sin(\phi/M) ,$$

with  $V_1$  large, of order  $M^4$ ,  $A$  and  $B$  much smaller, and  $M$  a large mass, but not larger than the Planck mass. This yields an *a priori* probability distribution (16) that varies appreciably in the anthropically allowed range of  $\phi$ .

It turns out<sup>18</sup> that the issue of whether the *a priori* probability (16) is flat in the anthropically allowed range of  $\phi$  depends on the way we impose the slow roll conditions (12). There is a large class of potentials for which the probability is flat in this range. Suppose for instance that, unlike the example chosen by Garriga and Vilenkin, the potential is of the general form

$$V(\phi) = V_1 f(\lambda\phi) \quad (17)$$

where  $V_1$  is a large energy density, in the range  $m_W^4$  to  $m_{\text{Planck}}^4$ ,  $\lambda > 0$  is a very small constant, and  $f(x)$  is a function involving no very small or very large parameters. Anthropically allowed values of  $\phi/\lambda$  must be near a zero of  $f(x)$ , say a simple zero at  $x = a$ . Then  $V'(\phi) \simeq \lambda V_1 f'(a) \approx \lambda V_1$  and  $V''(\phi) \simeq \lambda^2 V_1 f''(a) \approx \lambda^2 V_1$ , so both inequalities (12) are satisfied if

$$\lambda \ll \sqrt{8\pi G} \left( \frac{\rho_V}{V_1} \right) . \quad (18)$$

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<sup>18</sup>S. Weinberg: astro-ph/0002387.

Galaxy formation is only possible for  $|V(\phi)|$  less than an upper bound  $V_{\max}$ , of the order of the mass density of the universe at the earliest time of galaxy formation, which is very much less than  $V_1$ , so the anthropically allowed range of values of  $\phi$  is

$$|\phi - a/\lambda|_{\max} \simeq \frac{V_{\max}}{\lambda V_1 |f'(a)|}. \quad (19)$$

The fractional variation in the *a priori* probability density (16) as  $\phi$  varies in the range (19) is then

$$\left| \frac{V''(\phi)}{V'(\phi)} \right| |\phi - a/\lambda|_{\max} \simeq \left| \frac{V_{\max}}{V_1} \right| \left| \frac{f''(a)}{f'(a)^2} \right| \approx \left| \frac{V_{\max}}{V_1} \right| \ll 1 \quad (20)$$

justifying the assumptions made in the calculation of Eq. (10).

I should emphasize that no fine-tuning is needed in potentials of type (16). It is only necessary that  $V_1$  be sufficiently large,  $\lambda$  be sufficiently small, and  $f(x)$  have a simple zero somewhere, with derivatives of order unity at this zero. These properties are not upset if for instance we add a large constant to the potential. But why should each appearance of the field  $\phi$  be accompanied with a tiny factor  $\lambda$ ? As we have been using it, derivatives of the field  $\phi$  appear in the Lagrangian density in the form  $-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ , as shown by the coefficient unity of the second derivative in the field equation (1). In general, we might expect the Lagrangian density for  $\phi$  to take the form

$$\mathcal{L} = -\frac{Z}{2}\partial_\mu\phi\partial^\mu\phi - V_1f(\phi/M) \quad (21)$$

where  $f(x)$  is a function of the sort we have been considering, involving no large or small parameters,  $M$  is a mass perhaps of order  $(8\pi G)^{-1/2}$ , and  $V_1$  is a large constant, of order  $M^4$ . With an arbitrary field-renormalization constant  $Z$  in the Lagrangian, the field  $\phi$  is not canonically normalized, and does not obey Eq. (1). We may define a canonically normalized field as  $\phi' \equiv \sqrt{Z}\phi$ ; writing the Lagrangian in terms of  $\phi'$ , and dropping the prime, we get a potential of the form (16), with  $\lambda = 1/M\sqrt{Z}$ . Thus we can understand a very small  $\lambda$  if we can explain why the field renormalization constant  $Z$  is very large. Perhaps this has something to do with the running of  $Z$  as the length scale at which it is measured grows to astronomical dimensions.

There is a problem with this sort of implementation of the anthropic principle, that may prevent its application to anything other than the cosmological constant. When quantized, a scalar field with a very flat potential

leads to very light bosons, that might be expected to have been already observed. If we want to explain the masses and charges of elementary particles anthropically, by supposing that these masses and charges arise from expectation values of a scalar field in a flat potential with random initial values, then the scalar field would have to couple to these elementary particles, and would therefore be created in their collisions and decays. This problem does not arise for a scalar field that couples only to itself and gravitation (and perhaps also to a hidden sector of other fields that couple only to other fields in the hidden sector and to gravitation). It is true that such a scalar would couple to observed particles through multi-graviton exchange, and with a cutoff at the Planck mass the Yukawa couplings of dimensionality four that are generated in this way would in general not be suppressed by factors of  $G$ . But in our case the non-derivative interactions of the scalars with gravitation are suppressed by a factor  $V'(\phi) \propto \lambda$ , which according to Eq. (18) is much less than  $\sqrt{8\pi G}$ , yielding Yukawa couplings that are very much less than unity. Thus it may be that anthropic considerations are relevant for the cosmological constant, but for nothing else.